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1(a) (i) $s = ut + \frac{1}{2}ft^2$

Stage ab: $30 = u(4) + \frac{1}{2}f(16)$
$= 4u + 8f$

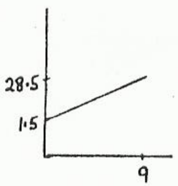
Stage ac: $84 = u(7) + \frac{1}{2}f(49)$
$= 7u + 24.5f$

$u = 1.5$ m/s and $f = 3$ m/s

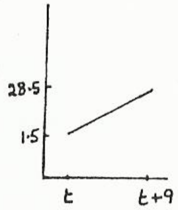
Stage ad: $s = ut + \frac{1}{2}ft^2$
$= 1.5(9) + \frac{1}{2}(3)(81)$
$= 135$ metres

(ii) Any two of the following points

0,1.5	t,1.5
4,13.5	t+4,13.5
7,22.5	t+7,22.5
9,28.5	t+9,28.5



OR



(b) (i) $s = ut + \frac{1}{2}ft^2$

P $s = 47(t+2) - 4.9(t+2)^2$

Q $s = 64.6t - 4.9t^2$

$s_P = s_Q$

$\rightarrow t = 2$ seconds

(ii) $s = ut + \frac{1}{2}ft^2$

$= 64.6t - 4.9t^2$

$= 64.6(2) - 4.9(4)$

$= 129.2 - 19.6$

$= 109.6$ metres

1993

193

2 (a) Let $\vec{V_w} = a\vec{i} + b\vec{j}$

$\vec{V_{wg}} = \vec{V_w} - \vec{V_g}$

from east $-x\vec{i} = a\vec{i} + b\vec{j} - (-11\vec{j})$

from south east $-y\vec{i} + y\vec{j} = a\vec{i} + b\vec{j} - (-22\vec{j})$

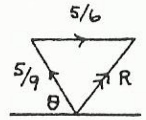
$b = -11$ and $a = -11$

$\vec{V_w} = -11\vec{i} - 11\vec{j}$

magnitude $= 11\sqrt{2}$ or 15.56 m/s

direction South West

(b) (i)



cross as quickly as possible when

$\frac{5}{9}\sin\theta$ is a maximum

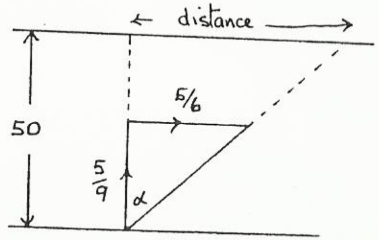
i.e. when $\sin\theta = 1$ or $\theta = 90°$

(ii) time to cross = distance / speed

$= \frac{50}{5/9}$ or $\left(\frac{90.139}{1.0015}\right)$

$= 90$ seconds

(iii)



$\tan\alpha = 5/6 \div 5/9$

$= 1.5$

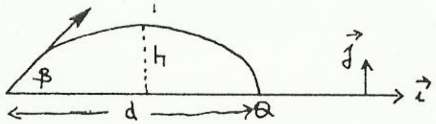
distance $= 50\tan\alpha$

$= 75$ metres

OR distance $= \frac{5}{6} \times 90 = 75$ m.

H93 1993

(a)



$\vec{r} = (u\cos\beta.t)\,\vec{i} + (u\sin\beta.t - 0.5gt^2)\,\vec{j}$

$\vec{v} = (u\cos\beta)\,\vec{i} + (u\sin\beta - gt)\,\vec{j}$

At P: $\quad \vec{v}_j = 0 \quad \Rightarrow \quad t = \dfrac{u\sin\beta}{g}$

$h = \vec{r}_j$

$= u\sin\beta.\dfrac{u\sin\beta}{g} - \dfrac{0.5g.u^2\sin^2\beta}{g^2}$

$= \dfrac{u^2\sin^2\beta}{2g}$

At Q: $\quad t = \dfrac{2u\sin\beta}{g}$

$d = \vec{r}_i$

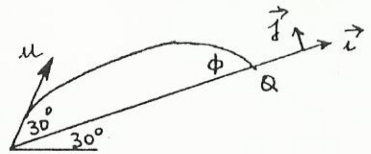
$= u\cos\beta.\dfrac{2u\sin\beta}{g}$

$= \dfrac{2u^2\sin\beta\cos\beta}{g}$

$d = 3h \quad \Rightarrow \quad \dfrac{2u^2\sin\beta\cos\beta}{g} = \dfrac{3u^2\sin^2\beta}{2g}$

$\Rightarrow \quad \tan\beta = 4/3$

(b) (i)



$\vec{r} = (u\cos30.t - 0.5g\sin30.t^2)\,\vec{i} + (u\sin30.t - 0.5g\cos30.t^2)\,\vec{j}$

$\vec{v} = (u\cos30 - g\sin30.t)\,\vec{i} + (u\sin30 - g\cos30.t)\,\vec{j}$

(ii) At Q: $\vec{r}_j = 0 \quad \Rightarrow \quad t = \dfrac{2u\sin30}{g\cos30}$

$= \dfrac{2u}{g\sqrt{3}}$

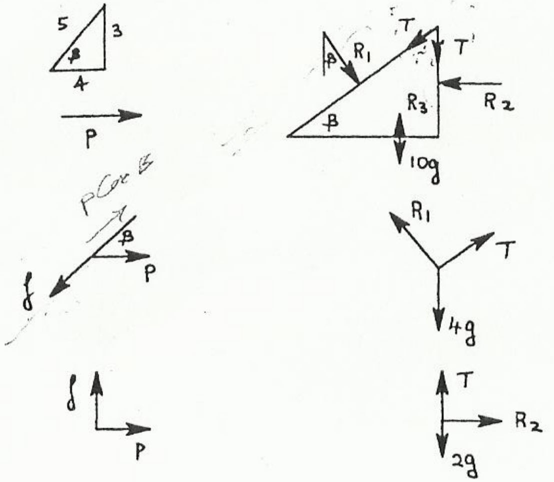
$\tan\phi = -\dfrac{\vec{v}_j}{\vec{v}_i}$

$= -\dfrac{(u\sin30 - 2u\sin30)}{u\cos30 - g\sin30.(2u/g\sqrt{3})}$

$\tan\phi = \sqrt{3}$ or $\phi = 60°$

4 (i)

H93



(ii) Wedge: horiz. $R_1\sin\beta - R_2 - T\cos\beta = 10p$ ....(1)

4 kg mass: $4g\sin\beta - T = 4(f - p\cos\beta)$ ....(2)

$4g\cos\beta - R_1 = 4p\sin\beta$ ....(3)

2 kg mass: horiz. $R_2 = 2p$ ....(4)

vert. $T - 2g = 2f$ ....(5)

Eliminate f from equations (2) and (5)

$4g\sin\beta - T = 2T - 4g - 4p\cos\beta$

$T = \dfrac{16p + 32g}{15}$ ....(6)

Substitute equations (3), (4) and (6) into equation (1)

$(4g\cos\beta - 4p\sin\beta)\sin\beta - 2p - \dfrac{(16p + 32g)}{15}\cos\beta = 10p$

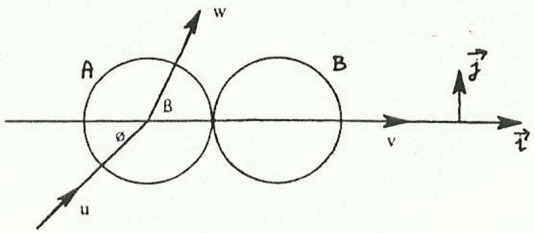
$p = g/67$ or 0.15

Laws. Principle of conservation of momentum
Newton's Experimental law

see 1990 solutⁿ or TEXTBOOK

P93

(i)



		vel before	vel after
A:	m	$u\cos\phi\,\vec{i} + u\sin\phi\,\vec{j}$	$w\cos\beta\,\vec{i} + u\sin\phi\,\vec{j}$
B:	m	$0\,\vec{i} + 0\,\vec{j}$	$v\,\vec{i} + 0\,\vec{j}$

PCM $\quad mu\cos\phi + 0 = mw\cos\beta + mv$

$$w\cos\beta = u\cos\phi - v$$

NEL $\quad v - w\cos\beta = -0.4(0 - u\cos\phi)$

$$w\cos\beta = v - 0.4u\cos\phi$$

$$\therefore\ u\cos\phi - v = v - 0.4u\cos\phi$$

$$v = 0.7u\cos\phi$$

$$= 0.7u\frac{5}{\sqrt{35}}$$

$$= \frac{u\sqrt{35}}{10}$$

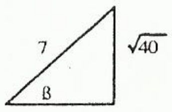
$$w\cos\beta = 0.3u\cos\phi$$

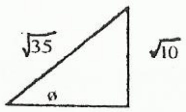
$$w(3/7) = 0.3u\frac{5}{\sqrt{35}} \Rightarrow w = \frac{u\sqrt{35}}{10} = v$$

(ii) KE before $= 0.5mu^2$

KE after $= 0.5mv^2 + 0.5mw^2 = mu^2(0.35)$

Loss of KE $= 0.5mu^2 - 0.35mu^2 = 0.15mu^2$ or $\frac{3mu^2}{20}$





6 (a) (i)

H93

no. 6

Force = mass x acceleration
$= m\omega^2 a$

$$\frac{7m}{16} = m\omega^2 a$$

$a = 4 \quad \Rightarrow \quad \frac{7}{16} = 4\omega^2$

$$\Rightarrow \quad \omega = \frac{\sqrt{7}}{8} \quad \text{or} \quad 0.33 \text{ rad/s}$$

$$T = \frac{2\pi}{\omega} = \frac{16\pi}{\sqrt{7}} \quad \text{or} \quad 19 \text{ seconds}$$

(ii)

$x = a\sin\omega t$

$= 4\sin\left(\frac{\sqrt{7}}{8}\cdot\frac{2\pi}{\sqrt{7}}\right)$

$= 2\sqrt{2}$

$v = \sqrt{a^2 - x^2}$

$= \frac{\sqrt{7}}{8}\sqrt{16 - 8}$

$= \sqrt{\frac{7}{8}}$ or 0.94 s

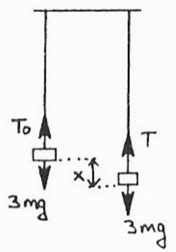
$v = a\omega\cos\omega t$

$= 4\frac{\sqrt{7}}{8}\cos\left(\frac{\sqrt{7}}{8}\cdot\frac{2\pi}{\sqrt{7}}\right)$

$= \frac{\sqrt{7}}{2}\cos(\pi/4)$

$= \frac{\sqrt{7}}{2\sqrt{2}}$ or $\sqrt{\frac{7}{8}}$

(b) (i)



$T_o = 3mg$

$ke = 3mg$

Force in direction of x increasing

$= 3mg - T$

$= 3mg - k(e + x)$

$= 3mg - ke - kx$

$= -kx$

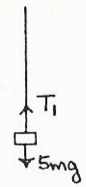
$= -\frac{48mg}{\ell}x$

Acceleration $= -\frac{16g}{\ell}x$

$\Rightarrow$ S.H.M. about $x = 0$ with $\omega = \sqrt{\frac{16g}{\ell}}$

Period $= \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\ell}{16g}}$ or $\frac{\pi}{2}\sqrt{\frac{\ell}{g}}$

(ii)



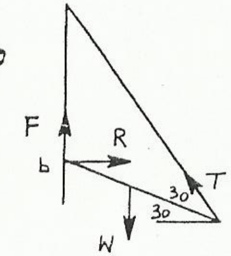
$T_1 = 5mg$

$ke + ky = 5mg$

$3mg + \frac{48mgy}{\ell} = 5mg$

amplitude $= y = \frac{\ell}{24}$

H93 1993 no. 7



moments about b:

$$T \sin 30 . 2\ell = W \sin 60 . \ell$$

$$T = \frac{W\sqrt{3}}{2}$$

horiz: $R = T\cos 60$ or $\frac{W\sqrt{3}}{4}$

vert: $\mu R + T \sin 60 = W$ or $F + T\sin 60 = W$

$$\mu . \frac{W\sqrt{3}}{4} + \frac{W\sqrt{3}}{2}.\frac{\sqrt{3}}{2} = W$$

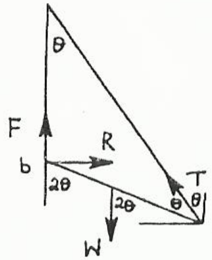
$$\mu = \frac{1}{\sqrt{3}}$$

| $F = W/4$

| Equilibrium if $F \le \mu R$

| $\mu \ge \frac{1}{\sqrt{3}}$

(ii)



moments about b:

$$T \sin\theta . 2\ell = W \sin 2\theta . \ell$$

$$T = W\cos\theta$$

horiz: $R = T\sin\theta$ or $W\sin\theta\cos\theta$

vert: $F + T\cos\theta = W$

$$F = W - W\cos\theta . \cos\theta = W\sin^2\theta$$

Rod slips if $F > \mu R$

$W\sin^2\theta > \mu W \sin\theta\cos\theta$

$\tan\theta > \mu$ or $\theta > 30°$

| $\frac{F}{R} = \frac{W\sin^2\theta}{W\sin\theta\cos\theta} = \tan\theta$

| $> \tan 30°$ (or μ)

8 (a) Let m = mass per unit length

mass of element = mdx

moment of inertia of element = $(mdx)x^2$

$$I = \int_{-\ell}^{\ell} mx^2 dx$$

$$= m\left[\frac{x^3}{3}\right]_{-\ell}^{\ell}$$

$$= \frac{2m}{3}\ell^3$$

$$= \frac{1}{3}M\ell^2 \quad \text{where } M = m.2\ell$$

(b) (i) $I = \frac{1}{3}m(0.6)^2 + m(0.2)^2$ or $0.16m$

Gain in K.E. = Loss in P.E.

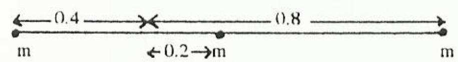
$$\frac{1}{2}I\omega^2 = mgh$$

$$0.08\, m\, \omega^2 = mg(0.4)$$

$$\omega = 7 \text{ rad/s}$$

$v = r\omega = 0.8(7) = 5.6$ m/s

(ii)



$I = 0.16m + m(0.8)^2 + m(0.4)^2$ or $0.96m$

$Mgh = mg(0.2) + mg(0.8) - mg(0.4)$ or $0.6mg$

$$T = 2\pi\sqrt{\frac{I}{Mgh}} = 2\pi\sqrt{\frac{0.96m}{0.6mg}} = 2.54 \text{ seconds}$$

9 (a)

H93

$$B = W$$

$$\frac{17W(1)}{20\,s} = W$$

$$s = \frac{17}{20}$$

no. 9

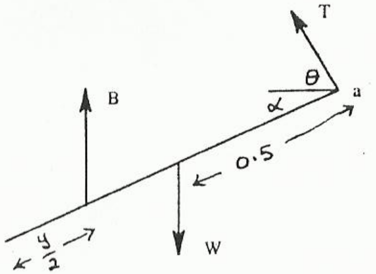
Let x = depth of layer of oil; A = cross-sectional area

$$B_{water} + B_{OIL} = W$$

$$1000A(20-x)g + 800Axg = 850A(20)g \quad \text{or} \quad \frac{\frac{(20-x)W(1)}{20}}{17/20} + \frac{\frac{xW(0.8)}{20}}{17/20} = W$$

$$x = 15 \text{ cm.}$$

(b) (i)



horiz: $T\cos\theta = 0 \Rightarrow \cos\theta = 0$

$\Rightarrow \theta = 90°$

(ii) $$B = \frac{y\,W(1)}{0.64}$$

moments about a:

$$W(0.5)\cos\alpha = B(1 - 0.5y)\cos\alpha$$

$$0.32 = y - 0.5y^2$$

$$y^2 - 2y + 0.64 = 0$$

$$(y - 0.4)(y - 1.6) = 0$$

$$y = 0.4 \text{ m.}$$

10 (a)

H93

1993

$$(x^2 + 2)\frac{dy}{dx} = x(y + 1)$$

$$\int \frac{dy}{y+1} = \int \frac{x.dx}{x^2 + 2}$$

$$\ln(y + 1) = 0.5\ln(x^2 + 2) + C$$

$y = 2, x = 1 \Rightarrow \ln 3 = 0.5\ln 3 + C$

$$C = 0.5\ln 3$$

$$\ln(y + 1) = 0.5\ln(x^2 + 2) + 0.5\ln 3$$

$$y + 1 = \sqrt{3x^2 + 6}$$

$x = 2 \Rightarrow y + 1 = \sqrt{18}$

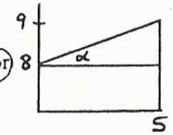
$y = 3\sqrt{2} - 1$ or 3.24

(b) (i) retardation $= 8 + kx$

$9 = 8 + k(5)$

$k = 0.2$

or



$\tan\alpha = 0.2$

$y = 0.2x + 8$

$$\frac{v\,dv}{dx} = -\left(8 + \frac{x}{5}\right)$$

(ii) $$\int_{20}^{0} v\,dv = -\int_{0}^{x_1}\left(8 + \frac{x}{5}\right)dx$$

$$\left[0.5v^2\right]_{20}^{0} = -\left[8x + \frac{x^2}{10}\right]_{0}^{x_1}$$

$$-200 = -8x_1 - 0.1x_1^2$$

$$x_1^2 + 80x_1 - 2000 = 0$$

$$(x_1 - 20)(x_1 + 100) = 0$$

$$x_1 = 20 \text{ metres}$$

[redacted]